of additional hydrogen in the vehicle's primary propulsion system for base drag reductions of as much as $\frac{1}{2}$.

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Measurements in a Free Piston Shock Tube

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EXPERIMENTAL measurements have been obtained in a free piston shock tube. This shock tube uses a piston compression as the method for transferring energy to the driver gas.[1, 2] The piston is propelled by high pressure, and the compression process continues until the diaphragm is broken. The compressed driver gas then expands into the expansion section driving a shock wave down the length of the tube.

The passage of the shock wave was detected by thin film resistance gages that were mounted in the shock tube wall. These gages were made by Rolf W. F. Gross. The Mach number of the shock wave was obtained from the resulting distance vs time measurements. The diameter of the expansion tube affects the performance of the shock tube because of the growth of the boundary layer on the wall behind the shock wave.[3–5] The smaller the tube diameter, the greater the attenuation of the shock speed, and the shorter the testing time. Our experiments were performed in a 2-in.-diam expansion tube. For Mach numbers less than 6 in helium, the maximum shock wave attenuation was $1\frac{1}{2}$%/ft length.

The speed of the reflected shock wave was obtained from the two successive signals produced by the thin film resistance gage nearest the end wall. The two signals correspond to the passage of the incident and the reflected shock wave, respectively. Knowing the distance between the gage and the end wall, we obtain the average speed of the two shock waves. Then, using the value for the speed of the incident shock wave, the speed of the reflected shock wave can be obtained. The results for helium are given in Table 1 and are in good agreement with those reported by Strehlow and Cohen[6] for argon. The theoretical values for the ratio $-U_R/U_S$ are bounded by

$$0.557 < -U_R/U_S < 0.567$$

for the Mach number range of Table 1.

The maximum available testing time in the region behind the first reflected shock wave, region 5 of Fig. 1, is also of interest. This is the time between the arrival of the incident shock wave at the end wall t_C and the arrival of the second reflected shock wave at the end wall t_E. The second reflected shock wave results from the interaction of the first reflected shock wave with the contact surface. The arrival times of these two shock waves were detected with a thin film re-

Table 1 Speed of reflected shock wave for helium

Incident shock wave	Ratio of speed of shock waves
M_s	$-U_R/U_S$
2.74	0.500
2.74	0.503
2.80	0.496
2.85	0.487
2.89	0.486
2.93	0.488
2.95	0.494

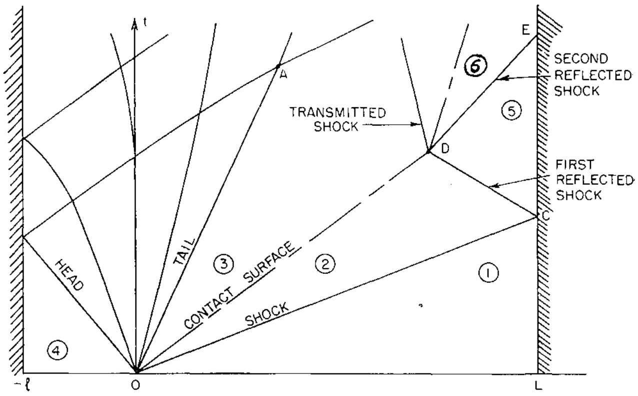

Fig. 1 Flow regimes in the shock tube.

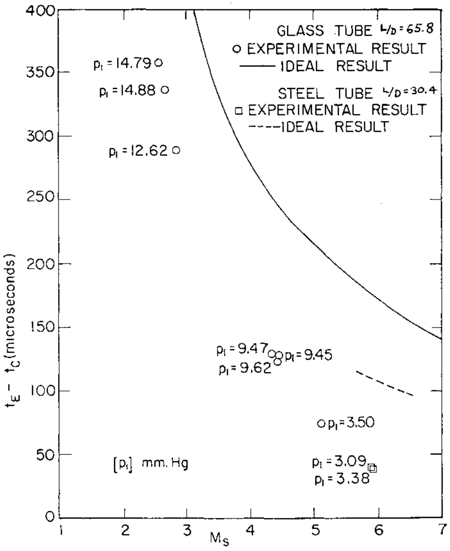

Fig. 2 Maximum testing time behind reflected shock wave.

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Table 2 Effect of expansion of driver gas through an area ratio of $\frac{1}{4}$

a_4/a_1	P_4/P_1	M_s	a_4'/a_1	P_4'/P_1	M_s'
3	155,000	9.3	2.5	62,000	7.6
4	155,000	11.7	3.3	62,000	9.7
5	155,000	13.7	4.2	62,000	11.5
6	155,000	15.6	5.0	62,000	12.9
8	155,000	19.0	6.7	62,000	15.6
10	155,000	22.1	8.4	62,000	17.9

sistance gage that was placed in the end flange. The resulting signals were displayed on an oscilloscope that permitted the determination of the maximum testing time $t_E - t_C$.

The experimental and ideal results are presented in Fig. 2. The latter were obtained as follows. Knowing the Mach number of the incident shock wave we may obtain the time of arrival of the shock wave at the end wall t_C and the time t_D when the shock wave, which is reflected from the end wall, encounters the contact surface. Then, if we know the velocity of the shock wave that is reflected from the contact surface (back to the end wall), we may obtain the quantity $t_E - t_D$. Adding this result to the quantity $t_D - t_C$ gives the ideal value for the maximum testing time.

To obtain the velocity of the second reflected shock wave, we must know the pressure ratio across the wave P_6/P_5. This may be determined by iteration.[7]

The inner diameter of the driver section was 4 in. To increase the driver gas temperature, the region adjacent to the diaphragm (on the upstream side) has a steel insert that reduces the cross section to a 1-in. square. When the diaphragm is broken, the driver gas expands from the 1-in. square section through a conically shaped transition piece to the 2-in.-diam expansion section. The expansion reduces the temperature and pressure of the driver gas, which results in a reduction in the Mach number of the shock wave. This effect was studied by Lin and Fyfe,[8] and we have used their results to obtain an estimate of the reduction in shock strength. The results are presented in Table 2 for an initial pressure ratio across the diaphragm P_4/P_1 of 155,000. The primed quantities represent the effective values for the driver gas for a driver-area to expansion-area ratio of $\frac{1}{4}$. The results indicate that the expansion of the driver gas through an area ratio of $\frac{1}{4}$ produces a significant reduction in Mach number.

A shock Mach number of 9.5 in helium (21,000 fps) has been attained in the Harvard free piston shock tube, a sufficiently strong shock to produce ionization in helium at a pressure of 1-atm behind the reflected shock (approximately 15,000° K according to equilibrium conditions).

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Mass Injection Contours for a Hypersonic Leading Edge at an Angle of Attack

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Mass injection through a rounded orifice at the stagnation point of a symmetric body at hypersonic speeds has been investigated experimentally by both Baron and Alzner[1] and Tucker,[2] and theoretically by Wang[3] and Eminton.[4]

The smoothly contoured orifice has the advantage of supplying a larger buffer distance at the points where heating is most serious. Moreover it can be treated theoretically such that the flow field is prescribed.

Most re-entry vehicles enter the atmosphere at some predetermined angle of attack. The present note calculates the asymmetric orifice contour for mass injection cooling for a parabolic leading edge at an angle of attack with respect to the freestream (Fig. 1).

A 'Newtonian' pressure distribution is imparted on the parabolic contact surface, which is inclined at an angle θ with the axis.

Using the coordinate system shown in Fig. 2, the Newtonian pressure is found to be

$$p = p_0 - \rho_\infty U_\infty^2 \sin^2[\tan^{-1}(R/y) - \theta - (\pi/2)] \quad (1)$$

where p_0 is the impact pressure at point A. This expression is valid along EAB where the surface is in the windward side of the freestream.

We transform to parabolic coordinates defined by

$$(x + iy) = (R/2) + (1/2R)(\xi + i\eta)^2 \quad (2)$$

From the equations of continuity

$$(\partial/\partial\xi)[(\xi^2 + \eta^2)^{(1/2)}\rho q_{(\xi)}] + (\partial/\partial\eta)[(\xi^2 + \eta^2)^{(1/2)}\rho q_{(\eta)}] = 0 \quad (3)$$

and momentum

$$(\partial/\partial\xi)[(\xi^2 + \eta^2)^{(1/2)}\zeta q_{(\xi)})] + (\partial/\partial\eta)[(\xi^2 + \eta^2)^{(1/2)}\zeta q_{(\eta)}] = 0 \quad (4)$$

one can obtain an equation for the stream function

$$\frac{1}{\rho}\frac{\partial}{\partial\xi}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial\xi}\right) + \frac{1}{\rho}\frac{\partial}{\partial\eta}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial\eta}\right) = -f(\psi)(\xi^2 + \eta^2) \quad (5)$$

where $\zeta = \rho f(\psi)$ is the vorticity.

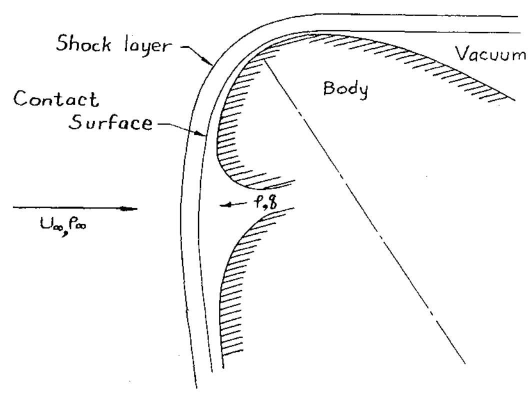

Fig. 1 Schematic diagram.

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